

Probing the nuclear EOS with fragment production

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We discuss fragmentation mechanisms and isospin transport occurring in central collisions between neutron rich systems at Fermi energies. In particular, isospin effects are analyzed looking at the correlations between fragment isotopic content and kinematical properties. Simulations are based on an approximate solution of the Boltzmann-Langevin (BL) equation. An attempt to solve the complete BL equation, by introducing full fluctuations in phase space is also discussed.

Key words: multifragmentation, isospin transport, symmetry energy, fluctuations, many-body approaches

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1 Introduction

In the last few years the increased accuracy of the experimental techniques has renewed interest in nuclear reactions at Fermi energies. Exclusive measurements, event-by-event analysis, and a 4π coverage allow a deeper investigation of the evolution of the reaction mechanisms with beam energy and centrality. New insights into the understanding of the nuclear matter equation-of-state (*EOS*) were gained [1]. In particular, recent experimental and theoretical analyses were devoted to the study of the properties and effects of the symmetry term of the *EOS* (iso-*EOS*) away from saturation conditions [2,3]. In particular, heavy ion reactions with exotic nuclei at Fermi energies can be used to study the properties of the symmetry term at densities below and around the normal value. In central collisions at 30-50 MeV/A, where the full disassembly of the system into many fragments is observed, one can study specifically properties of liquid-gas phase transitions occurring in asymmetric matter [3,4,5,6]. For instance, in neutron-rich matter, phase co-existence leads to a different asymmetry in the liquid and gaseous phase: fragments (liquid) appear more

symmetric with respect to the initial matter, while light particles (gas) are more neutron-rich. Hence the analysis of the isotopic content of all reaction products, from pre-equilibrium emission to fragments, allows to get information on low-density properties of the isovector part of the nuclear interaction.

In recent years, the properties of fragments and light clusters emitted in systems with different initial asymmetries have been widely investigated [7,8,9,10,11,12,13], looking in particular at the production yields of various isotopes, as obtained in reactions between proton-rich and neutron-rich systems.

More recently, the study of the isotopic content of pre-equilibrium emission has revealed a good sensitivity to the iso-EOS, considering the emitted neutron to proton ratio as a function of the kinetic energy [14]. Here we extend this type of investigation to fragments. Correlations between fragment charges and velocities have been recently observed, providing information on the interplay between thermal and entrance channel (collective) effects in the fragmentation mechanism [16,17]. Following this line, one can also investigate correlations between fragment isotopic content and kinematical properties, trying to get a deeper insight into the reaction path and to improve the understanding of the underlying mechanisms. In this way one can also study more in detail the effects of different EOS's and, in particular, of the symmetry energy on fragment properties [18].

The collision dynamics is described on the basis of approximate treatments of the Boltzmann-Langevin (BL) approach, i.e. of transport equations including fluctuations. In particular, we will adopt the Stochastic Mean Field (SMF) method, where fluctuations are projected onto the ordinary space [19].

A new method to solve the complete BL equation, by implementing the full fluctuations in phase space will be discussed in the last part of the manuscript (Section 4).

2 The model

Theoretically the evolution of complex systems under the influence of fluctuations can be described by a transport equation with a fluctuating term, the so-called Boltzmann-Langevin equation (BLE):

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\} = I_{coll}[f] + \delta I[f], \quad (1)$$

where $f(\mathbf{r}, \mathbf{p}, t)$ is the one-body distribution function, $H(\mathbf{r}, \mathbf{p}, t)$ is the one-body Hamiltonian and $\delta I[f]$ represents the fluctuating part of the two-body

collision integral [20,21].

Here we will follow the approximate treatment to the BLE presented in Ref.[19], the so-called Stochastic Mean Field (SMF) model, that consists in the implementation of spatial density fluctuations.

Calculations have been performed using the TWINGO code, where the test particle method is used to solve Eq.(1) [22]. We adopt a soft EOS, with compressibility modulus $K = 200 \text{ MeV}$ and, for the density (ρ) dependence of the symmetry energy, we consider two representative parameterizations, $E_{sym}(\rho, I)/A \equiv C_{sym}(\rho)I^2$, $I \equiv (N - Z)/A$: one showing a rapidly increasing behaviour with density, roughly proportional to ρ^2 (asystiff) and one with a kind of saturation above normal density (asysoft, SKM^*) (see Ref.s[3,23] for more detail). The two parameterizations obviously cross at normal density. The symmetry energy at densities below the normal value is larger in the asysoft case, while above normal density it is higher in the asystiff case. Hence in the low-density regime, that is the region of interest for our analysis, isospin effects are expected to be stronger in the asysoft case.

3 Fragment isotopic properties and correlations

We will focus on central collisions, $b = 2 \text{ fm}$, considering symmetric reactions between systems having three different initial asymmetry: $^{112}\text{Sn} + ^{112}\text{Sn}$, $^{124}\text{Sn} + ^{124}\text{Sn}$, $^{132}\text{Sn} + ^{132}\text{Sn}$, with $(N/Z)_{in} = 1.24, 1.48, 1.64$, respectively. The considered beam energy is 50 MeV/A . 1200 events have been run for each reaction and for each of the two symmetry energies adopted.

The first two reactions, $^{112}\text{Sn} + ^{112}\text{Sn}$ and $^{124}\text{Sn} + ^{124}\text{Sn}$ have been widely investigated both from the experimental and theoretical point of view [7,23,24,25]. In central collisions, after the initial collisional shock, the system expands and breaks up into many pieces, due to the development of volume (spinodal) and surface instabilities. The formation of a bubble-like configuration is observed, where the initial fragments are located. The average fragment multiplicity is approximately equal to 6 for the reactions considered here [23]. Along the reaction path, several nucleons are emitted at the early stage (pre-equilibrium emission) and/or are evaporated while fragments are formed. Primary fragments are identified by applying a coalescence procedure to the matter with density larger than $\rho_{cut} = 1/5 \rho_0$ (liquid phase). The remaining particles are considered as belonging to the gas phase.

First, let us briefly recall some general features concerning the isotopic content of fragments and emitted nucleons, as obtained with the two iso-EOS's considered. In the following we will restrict our analysis to fragments with charge in

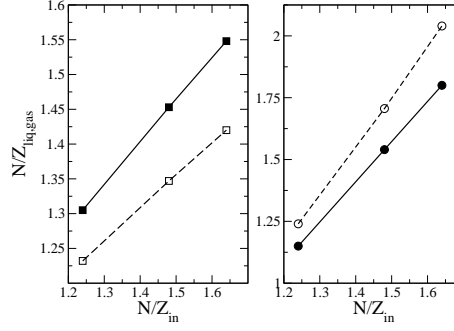


Fig. 1. The N/Z of the liquid (left) and of the gas (right) phase is displayed as a function of the system initial N/Z . Full lines and symbols refer to the asystiff parameterization. Dashed lines and open symbols are for asysoft.

the range between 3 and 10 (that we call intermediate mass fragments (IMF's)). The average N/Z of emitted nucleons (gas phase) and IMF's is presented in Fig.1 as a function of the initial $(N/Z)_{in}$ of the three colliding systems. One observes that, generally, the gas phase (right panel) is more neutron-rich in the asysoft case, while IMF's (left panel) are more symmetric. This is due to the larger value of the symmetry energy at low density for the asysoft parameterization [23]. It is interesting to note that, in the asystiff case, due to the rather low value of the symmetry energy, Coulomb effects dominate and the N/Z of the gas phase becomes lower than that for IMF's, because protons are preferentially emitted. Now we move to investigate in more detail correlations between fragment isotopic content and kinematical properties. The idea in this investigation is that fragmentation originates from the break-up of a composite source that expands with a given velocity field. Since neutrons and protons experience different forces, one may expect a different radial flow for the two species. In this case, the N/Z composition of the source would not be uniform, but would depend on the radial distance from the center or mass or, equivalently, on the local velocity. This trend should then be reflected in the fragment asymmetries. As a measure of the isotopic composition of the IMF's, we will consider the sum of neutrons, $N = \sum_i N_i$, and protons, $Z = \sum_i Z_i$, of all IMF's in a given kinetic energy range, in each event. Then we take the ratio N/Z and we consider the average over the ensemble of events. This observable is plotted in Fig.2 for the three reactions. The behaviour observed is rather sensitive to the iso-EOS. For the proton-rich system, the N/Z decreases with the fragment kinetic energy, especially in the asystiff case, where the symmetry energy is relatively low at low density [23]. In this case, the Coulomb repulsion pushes the protons towards the surface of the system. Hence, more symmetric fragments acquire larger velocity. The same effects are responsible for the proton-rich pre-equilibrium emission observed in this case (see Fig.1). The decreasing trend is less pronounced in the asysoft case (right panel) because Coulomb effects on protons are counterbalanced by the larger attraction

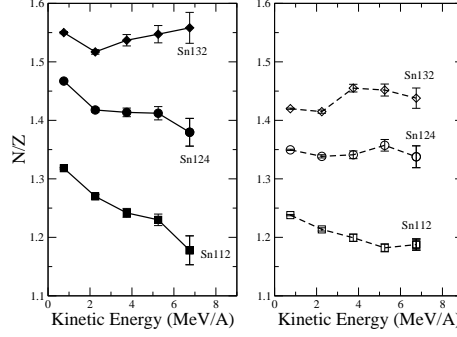


Fig. 2. The fragment N/Z (see text) as a function of the kinetic energy. Left panel: asystiff; Right panel: asysoft.

of the symmetry potential. In systems with higher initial asymmetry, the decreasing trend is inverted, due to the larger neutron repulsion in neutron-rich systems. Larger slopes are always observed in the asysoft case.

In conclusion, this analysis reveals the existence of significant, EOS-dependent correlations between the N/Z and the kinetic energy of IMF's. This correlation is linked to the different forces experienced by neutrons and protons along the fragmentation path, that in turn depend on the detail of the isovector part of the nuclear interaction. This study can be considered as complementary to pre-equilibrium emission studies [14,15]. A parallel investigation of pre-equilibrium and fragment emissions would be very important for a cross-check of model predictions against experimental observables sensitive to different phases of the reaction.

4 A new way to implement the full BL equation

The results discussed above were based on an approximate treatment of the BL equation, where only fluctuations in the ordinary space are considered. This is justified by the fact that the multi-fragmentation mechanism is mostly dominated by spatial density fluctuations. However, a more accurate representation of the full phase space dynamics would improve the description of fluctuations of fragment kinematical properties, allowing to investigate more sophisticated observables and to study the fragmentation path in deeper detail.

Within the test particle method, that is usually adopted to solve numerically the BNV equation, each nucleon is represented by a collection of N_{test} test particles, that are propagated according to the mean-field interaction and hard two-body scattering. Since collisions are treated stochastically for the single test particles, the dispersion around the average number of nucleon collisions is automatically divided by N_{test} . Hence fluctuations are largely reduced.

To overcome this problem, Bauer et al. [26] proposed a method to introduce a correlation between close particles in phase-space. The method follows the idea, first applied in extended TDHF, of evolving the single-particle density including a perturbative mixing of Slater determinants. This is realized in the collision integral by means of the following procedure: The nucleon-nucleon (NN) cross section is reduced by a factor N_{test} . Two test particles i_1 and i_2 are chosen as colliding partners, and will be moved to new positions in \mathbf{p} space provided that the corresponding transition probability is larger than a random number (extracted between 0 and 1), as usually done in transport codes using the test particle method. If the two particles can collide, i.e. the final positions are not forbidden by the Pauli blocking, the scattering actually involves two “clouds” of neighbouring test particles, corresponding to two entire nucleons ($2 \times N_{test}$ particles), that are chosen as the particles at closest distance in phase space.

This method is conceptually simple, and moreover the implementation into existing transport codes is really straightforward. However, since the Pauli-blocking is only checked for the first two colliding test particles and not for the entire swarm of particles that are actually moved, at the end the fluctuation strength does not reproduce the expected value for fermionic systems, but approaches instead the classical value.

A careful modification of the original procedure can considerably improve the results. The procedure that we propose can be summarized in the following steps:

- (i) The choice of the two colliding partners closely traces the standard recipe. If two test particles are allowed to collide, moving from the initial positions $\mathbf{p}_1, \mathbf{p}_2$ to new positions, $\mathbf{p}_3, \mathbf{p}_4$ corresponding to a rotation by (θ, ϕ) with respect to the relative momentum direction, two clouds of N_{test} particles are moved.
- (ii) Only particles within a sphere, in coordinate space, around the center of mass of the two partners i_1 and i_2 can belong to the clouds. The distance criterion is $|\mathbf{r}_i - \mathbf{r}_{CM}(i_1, i_2)| < d_r$, where d_r is a free parameter (see later discussion). Then a grid is introduced in momentum space, the size of each cell being V_{cell} .
- (iii) indicating by I and J the cells containing the partners i_1 and i_2 , we consider the corresponding final cells I' and J' in the frame rotated by (θ, ϕ) . For a given set of initial and final cells, the number of test particles that will be actually moved to final states is equal to the minimum between the occupation of the initial cells and the availability of the final ones $\bar{n}' = (1 - n')$:

$$n_t(I, I'; J, J') = \min(n_I, n_J, \bar{n}_{I'}, \bar{n}_{J'})$$

- (iv) Surrounding cells are explored with the same prescriptions, until one entire nucleon is reconstructed around i_1 and i_2 . The search procedure is

symmetric with respect to the center of mass of the two partners.

- (v) A further check on the total momentum of the two clouds is performed, and the origin of the grid is eventually slightly displaced in order to have a perfect energy and momentum conservation.

This method involves two parameters, namely the radius of the sphere in \mathbf{r} space, d_r , and the size of the momentum cells, V_{cell} . The radius d_r fixes the spatial extension of the nucleon, that in turn influences its spreading in momentum space. Hence this parameter fixes somehow the extension of the nucleon wave packet in phase-space and generally could affect the transport dynamics. It can be constrained by physical arguments, and in general it depends on physical properties of the system, such as its dilution. The cell size V_{cell} should be small enough to allow an accurate check of the Pauli blocking, but large enough to contain a sufficient number of test particles to reduce numerical uncertainties.

4.1 *Check of fluctuations*

We discuss our procedure to build fluctuations in the context of a system of fermions at equilibrium, at a given density and temperature, i.e. initialized according to a Fermi-Dirac distribution. This is a situation that is easily reached in the course of a nuclear reaction, after the initial collisional shock.

We emulate a piece of nuclear matter by taking a box with periodic boundary conditions. Since we will focus on fluctuations in momentum space, only one cell is present in coordinate space, and all particles can be chosen to collide (no restrictions in \mathbf{r} space). The size of the box is $l = 26 \text{ fm}$, and we put 2820 nucleons, so that the density has the saturation value $\varrho_0 = 0.16 \text{ fm}^{-3}$; each nucleon is represented by a collection of 500 test particles. Besides, we do not consider any distinction between neutrons and protons, so that one nucleon occupies at least a phase-space volume $h^3/4$. We initialize the momenta so as to reproduce a Fermi-Dirac profile corresponding to a temperature of 5 MeV ; finally, we consider a constant cross section $\sigma = 160 \text{ mb}$. In these calculations the volume V_{cell} corresponds to a cube of size $l_s = 30 \text{ MeV}/c$ (and coincides with $V_p = h^3/(4V_r)$). Calculations are stopped when the fluctuation variance saturates.

First of all, we have checked that our fluctuating collision integral preserves the average evolution of the system, i.e. does not change the one-body density profile. Also the average collision rate is in good agreement with analytical expectations [27].

One can directly visualize the effect of the fluctuating term by selecting a thin region in momentum space around the Fermi momentum. This roughly

corresponds to look at a bi-dimensional system. For this purpose, we adopt a new set of coordinates $(dp, pd\theta, p \sin(\theta)d\phi)$; for fixed p and dp , the distribution function will depend only on two coordinates, namely:

$$f(p, \theta, \phi) \rightarrow f(\theta, \eta)$$

with $d\eta = \sin(\theta)\phi$. We choose $p = 260 \text{ MeV}/c$ (approximately equal to the Fermi momentum) and $\Delta p = 10 \text{ MeV}/c$. In Fig. 3 we plot the distribution function $f(\theta, \eta)$ at two different times (initial and final times). At the initial time, f is nearly uniform, and its fluctuations are simply due to the numerical noise induced by the finite number of test particles. At later times, we observe the growth of fluctuations, evidenced by the typical structure with “peaks and holes”.

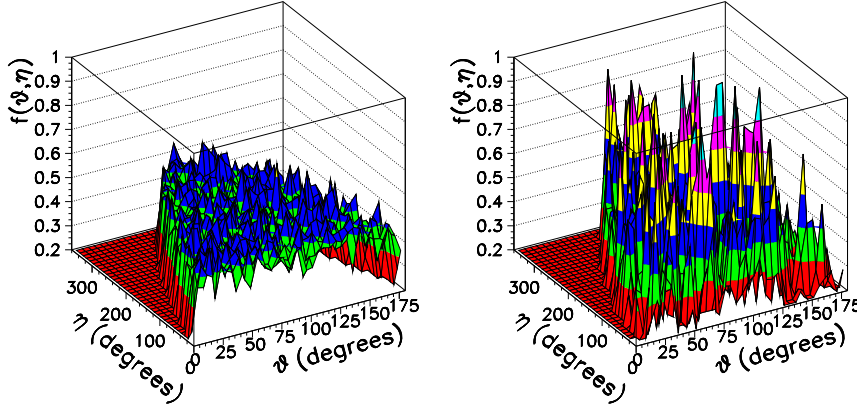


Fig. 3. Distribution function $f(\theta, \eta)$ at time $t = 0$ (left) and $t = 100 \text{ fm}/c$ (right).

In order to check the amplitude of fluctuations built by our procedure, we construct the density variance in a phase-space cell of volume V , $\sigma^2 = \langle \delta f(\mathbf{p}) \delta f(\mathbf{p}) \rangle$. At the Fermi surface, the expected value of the variance is equal to: $\sigma^2 = f_{eq}(1 - f_{eq})/N_V = 0.25/N_V$, where N_V is the number of nucleons that can be contained, at most, inside the volume V : $N_V = 4V/h^3$. f_{eq} denotes the average equilibrium value of the distribution function that, at the Fermi surface, is equal to 0.5.

Since the nucleon wave packet is generally deformed, it extends over volumes larger than $h^3/4$, hence the correct value of fluctuations is recovered in the “continuum limit”, i.e. in large volumes V , as shown in Fig.4. The slight deviation from the expected value, 0.25, is due to some difficulties, inherent to the adopted numerical procedure, in reconstructing entire nucleons in all collisional events.

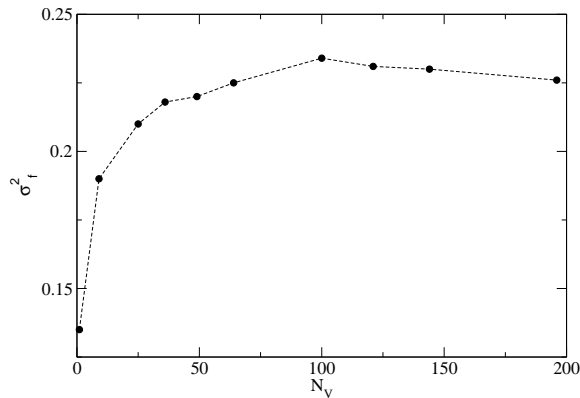


Fig. 4. Variance of the distribution function (rescaled by N_V) as a function of the number N_V of nucleons contained in the cell considered to construct the fluctuation variance.

5 Conclusions

We have discussed properties of the fragmentation path in central collisions of charge-asymmetric systems, that can be related to the behaviour of the symmetry energy below normal density, thus allowing to extract information on fundamental quantities of the nuclear interaction. We focused on the analysis of correlations between fragment isotopic content and kinetic energy, performing simulations based on SMF approaches. An EOS-dependent relation between these two observables is found. This study also allows one to get a deeper insight into the fragmentation mechanism. In fact, the analysis of correlations between fragment composition and velocity can be used as a clock of fragment formation and as an indicator of the underlying dynamics.

A new method to improve the treatment of fluctuations in transport approaches is also discussed. This is relevant not only for fragmentation studies, but in general for the dynamical description of quantum many-body systems.

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